

## The flow around a surface-mounted cube in uniform and turbulent streams

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An experimental investigation of the flow around surface-mounted cubes in both uniform, irrotational and sheared, turbulent flows is described. The shear flow was a simulated atmospheric boundary layer with a height ten times the body dimension. Measurements of body surface pressures and mean and fluctuating velocities within the wake are presented. In the latter case a pulsed-wire anemometer was used extensively since the turbulent intensities were much too high for effective use of more standard instrumentation. The clear effects of upstream turbulence and shear on the wake flow are described, comparisons with the somewhat sparse measurements of previous workers are made and the relevance of recent theoretical attempts to describe the flow, as opposed to numerical calculation techniques to predict it, is briefly discussed.

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### 1. Introduction

Arising from the increasing practical importance of bluff-body aerodynamics, there has been, over the past few decades, an enormous increase in the literature concerning laboratory simulations, full-scale measurements and, more recently, numerical calculations and theoretical predictions of a wide variety of bluff-body flows. It is significant that, although most of the real-life flows that fall into this general category are highly complex, the great majority of reported experiments are of a much simpler and more fundamental nature. Such work is, of course, vital to a proper understanding of the more complex situations and much remains to be done before even the simpler flows are understood. However, it is surprising that attention has been concentrated almost entirely on the flow around two-dimensional bodies. Even here only comparatively recently have the important effects of upstream turbulence and shear been demonstrated quantitatively. As far as surface-mounted bodies are concerned the emphasis has been on 'fence' flows (e.g. Good & Joubert 1968; Plate 1971), with the associated practical problem of windbreaks and shelter belts (Jensen 1954).

By contrast, flows behind full-scale or model buildings have been investigated much less thoroughly, despite the obvious practical implications regarding, say, effluent dispersion or wind loads on other buildings. Indeed, apart from some relatively sparse surface pressure measurements and general flow-visualization studies (e.g. Hamilton 1962; McLaren 1970) there is very little experimental data available even for the simpler case of surface-mounted bodies in a uniform

upstream flow. (By a 'uniform upstream flow', we mean one in which the mean velocity is uniform and the turbulence intensity is very low—less than, say, 0.5%—except in the thin boundary layer which must exist on the surface even if the body is mounted on a false floor.) From a practical point of view the wake, particularly that part of it close to the body, is of some importance; indeed, in principle, it should be investigated as thoroughly as the surface pressure field if any proper understanding of the total flow is to be obtained. With the advent of comprehensive numerical calculation techniques for turbulent flows some workers are already attempting to calculate 'simple' bluff-body flows: even three-dimensional ones (Hirt & Cook 1972). Others are formulating theoretical models of the flow (Counihan, Hunt & Jackson 1974; Hunt 1970). The success of either of these approaches can be finally assessed only by direct comparisons with experimental data but such comparisons for three-dimensional flows at least are hampered by the distinct lack of such data.

With these thoughts in mind, an experimental investigation of the flow around a surface-mounted cube, first in a uniform upstream flow and then in a simulated atmospheric boundary layer, has been undertaken. Measurements of mean surface pressures on the body and mean and fluctuating velocities in the wake have been made; these are presented in § 4. The highly turbulent nature of the near wake has in the past been a major source of experimental difficulty and, no doubt, one of the reasons why so few quantitative studies have been made. However, the pulsed-wire anemometer, fully described by Bradbury & Castro (1971) and developed considerably since then, is capable of making fairly accurate measurements in such highly turbulent regions, and one of the interesting features of the present work is felt to be the clear demonstration of its usefulness in these situations.

To highlight some of the major features of the flow subsidiary measurements are presented in § 5, where the wake flow is discussed. These include measurements of the swirl velocities in the wake and measurements of surface pressures for a range of ratios of cube height to boundary-layer height. It is felt that these measurements, together with those presented in § 4, provide a fairly clear picture of the wake flow and the way that this is affected by upstream turbulence and shear. During the course of the work a large quantity of data was accumulated and of necessity it cannot all be presented here. What we have tried to do is to extract sufficient data to demonstrate the main features of the flow. For further understanding of the various mechanisms involved we feel that studies of, for example, the separate effects of turbulence and shear would be desirable.

Comparison with previous data is made where possible and § 5 includes reference to some of the various numerical and theoretical attempts being made to calculate such flows. The difficulties in both approaches are emphasized and it is seen that, whilst these are considerable, such work is important and must be complemented by further systematic and thorough experimental work.

## 2. Apparatus and techniques

All the measurements in the case of the uniform upstream flow were made in an open-circuit wind tunnel with a working-section area of  $0.27 \times 0.90$  m. For preliminary measurements a series of wooden cubes was mounted on the tunnel floor itself, where the floor boundary layer was about 6 mm thick, but for the majority of the measurements a fully pressure-tapped brass cube of side 60 mm was mounted on a ground plate about 20 mm from the tunnel floor. The boundary-layer thickness on the ground plate at the cube position was less than 1.5 mm, the velocity in the empty tunnel was uniform to within  $\pm \frac{1}{4}\%$  over the contraction exit and the free-stream longitudinal turbulent intensity was less than about  $\frac{1}{2}\%$ .

Most of the measurements made with a thick upstream boundary layer were undertaken in a  $2.7 \times 9.1$  m wind tunnel usually used for investigations of effluent diffusion from particular sources, involving large-scale models of the site and its surrounding topography at scales between 1:1000 and 1:300. The simulation of the neutrally stable atmospheric boundary layer for these investigations is based on the system originally described by Counihan (1969) and consists basically of a series of vorticity generators mounted across the tunnel, with a relatively small barrier wall mounted upstream and distributed roughness elements downstream. In the present case use was made of the system which generated a boundary layer 2 m thick.

The cube position was  $3.5\delta$  (boundary-layer heights) downstream of the vorticity generators and the profiles of mean velocity, shear stress and turbulent intensity at this station are shown in figures 1 (*a*) and (*b*). They are a little different from the profiles at the end of the working section, a further  $4.5\delta$  downstream. The consequent longitudinal gradients were at worst about 3% per boundary-layer thickness, so since the cube height (200 mm) was only  $\frac{1}{10}\delta$  the effect of these gradients on the flow around the cube was undoubtedly very small. The roughness length  $y_0$  was about 0.02% of the boundary-layer height, so that  $y_0/h$  was about 0.02. A complete description of the boundary-layer characteristics can be found in Castro, Jackson & Robins (1975).

In addition to the measurements made in these two flows some less comprehensive tests were done in the  $0.27 \times 0.91$  m tunnel using a series of cubes (20, 40, 60 and 80 mm) mounted in a variety of thick boundary layers generated in a similar way to the 2 m boundary layer described above;  $\delta/h$ , where  $h$  is the cube height, varied from about 0.43 to 10. The boundary layers were less well investigated than the flow 2 m thick but the salient details will be described where necessary.

Apart from the pulsed-wire anemometer instrumentation was fairly standard. Where hot-wire anemometry was used frequent calibrations were made, particularly in the large tunnel, which draws air direct from the atmosphere. Pressure measurements were made by using either a multibank inclined manometer (in the case of the uniform upstream flow) or a capacitance micromanometer whose electrical output was integrated by an analog computer or a true integrating voltmeter. The pulsed-wire anemometer was interfaced directly to a desk-top

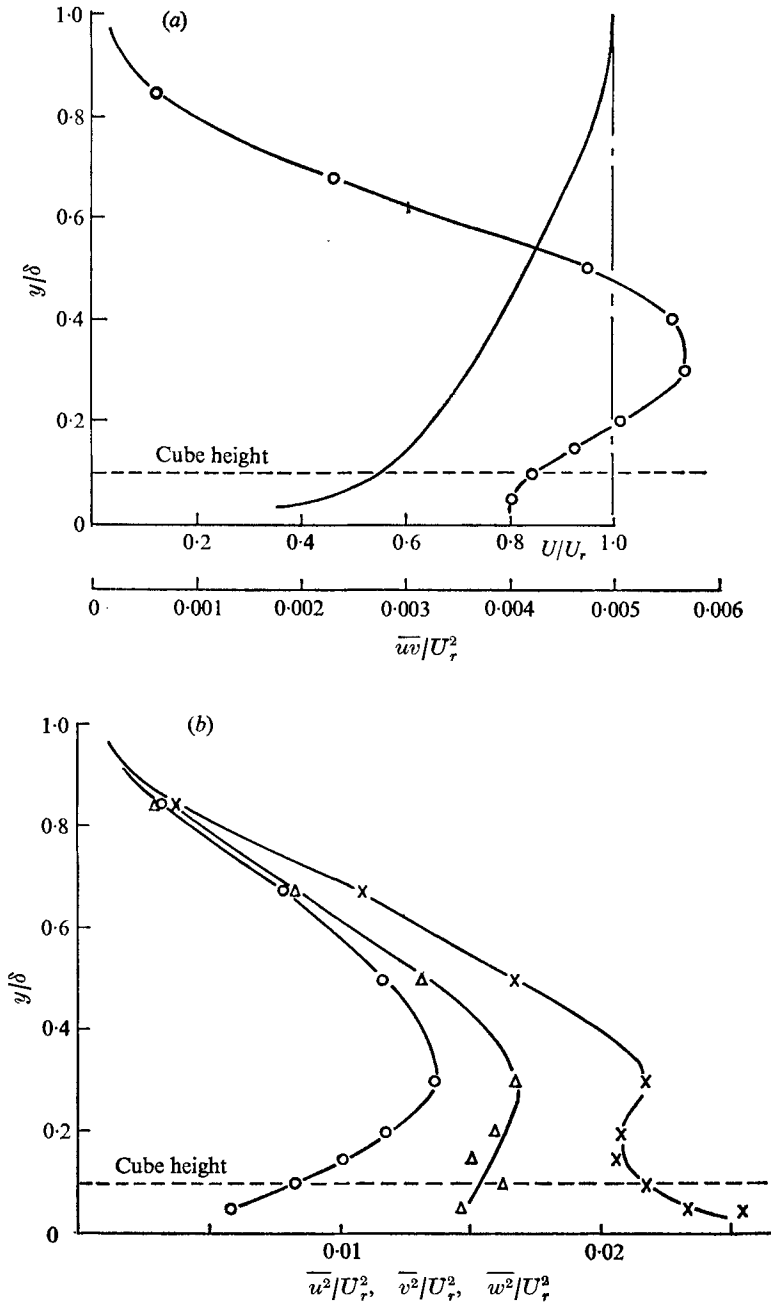


FIGURE 1. Measurements in the 2m simulated atmospheric boundary layer used for case B. (a) —, mean velocity; —○—, shear stress. (b) Normal turbulent stresses: x  $\overline{u^2}/U_r^2$ ; ○,  $\overline{v^2}/U_r^2$ ;  $\Delta$ ,  $\overline{w^2}/U_r^2$ .

calculator or a mini-computer, either of which could be programmed to perform calibration routines or the actual data collection and analysis.

For measurements of the transverse components of the mean velocity (in the case of the uniform upstream flow) a five-hole pitch-and-yaw tube was used. This had an external diameter of 2 mm and with careful calibration could measure flow angles as small as  $\frac{1}{4}^\circ$ . In a sheared turbulent flow the response of such an instrument is somewhat uncertain but most of the measurements were made in regions where the longitudinal turbulent intensity was less than about 15 %.

### 3. Preliminary measurements

Figure 2 defines the notation used throughout the rest of the paper. Since flow separation occurs at the leading edges of the cube it was thought unlikely that there would be any serious Reynolds number effects in the range of interest. In the case of the uniform upstream flow, hereafter referred to as case *A*, two distinct Reynolds number effects were, however, noticed. With the cube normal to the flow it was clear that as the Reynolds number based on free-stream velocity and cube height increased, the shear layer separating from the top of the cube moved upwards, away from the surface. This was undoubtedly a result of the transition point moving nearer to the start of the shear layer (as was demonstrated by hot-wire traces), but beyond a Reynolds number of about  $3 \times 10^4$  the shear layer appeared to be turbulent right from the leading edge and no further variations took place. Second, with the cube at  $45^\circ$  to the approach flow the high negative pressure measured near the top leading corner increased in magnitude by a factor of three as the Reynolds number increased from about  $2 \times 10^4$  to  $1 \times 10^5$ . On the other hand the base pressure (measured at the centre of one of the rear faces) remained constant. Leutheusser & Baines (1967) reported similar Reynolds number effects. The pressure gradients near the leading edges are, as will be shown in § 4, very high, so that only a very small shift in the pressure field would be necessary to produce such large changes in the pressure measured at a particular location in this region. Such variations in the pressure field could be caused by, for example, changes in the nature of the boundary layers on the front faces of the cube or in the position of the front stagnation point, which cannot be exactly on the vertical edge of the body since this would be an inherently unstable situation (see § 4). Alternatively the large variations in measured pressure might possibly be caused by static-hole Reynolds number effects, although this seems improbable. Since the base pressure did not change it seemed likely that the general wake structure did not change either although direct observations were not made. However, it is clear that statements regarding the irrelevance of the Reynolds number need to be made with some caution. All the pressure measurements described in § 4.1 were made at Reynolds numbers greater than  $10^5$  and the wake measurements (§ 4.2) at Reynolds numbers greater than  $3 \times 10^4$ . For the case of a turbulent upstream boundary-layer flow, case *B*, no Reynolds number effects were discernible for free-stream (gradient) velocities above about 0.5 m/s (corresponding to a Reynolds number based on cube height

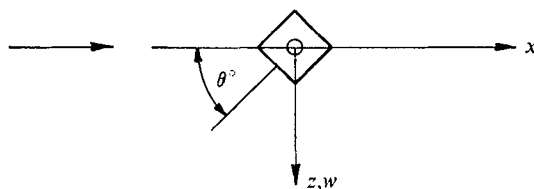
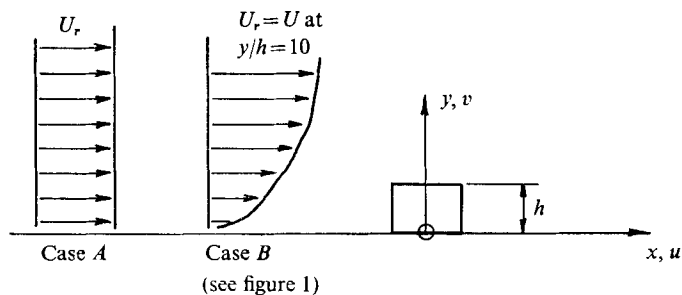


FIGURE 2. Notation.

and the velocity at that height in the undisturbed flow of about  $4 \times 10^3$ ) as might be expected.

A more serious problem associated with wind-tunnel tests on the flow around bluff bodies is that of blockage. In case *B* the blockage ratio, defined as the ratio of the frontal area of the body to the tunnel cross-sectional area, was only about 0.002, which is usually considered to be an insignificant value. However, in the  $0.27 \times 0.91$  m tunnel, even a 40 mm cube gives a blockage ratio of about 0.007. Some vertical profiles of mean velocity and turbulent intensity were measured at two downstream stations for three cube sizes, 40, 60 and 80 mm. Whilst the intensities collapsed reasonably well there was a definite trend in the mean velocity results. The effect of the tunnel walls was of course to 'constrict' the wake and this effect clearly increased with increasing blockage. There were, no doubt, corresponding differences in the pressure distributions and hence the drag. Since there is no really adequate theory for blockage effects on three-dimensional surface-mounted bodies, no attempt has been made to correct the results, so that particular care must be taken in any quantitative comparison with other work. To minimize blockage effects whilst ensuring a reasonable physical size for the experiments 60 mm cubes were used for the bulk of the work described in the following sections (for case *A*). The corresponding blockage ratio was therefore about  $1\frac{1}{2}\%$ .

To obtain some idea of the flow in case *A*, surface flow patterns were produced by standard oil flow techniques. Figure 3 (plate 1) shows the patterns obtained for the two symmetric cube orientations and whilst it is often dangerous to draw detailed conclusions from the surface flow patterns, some general observations can be made. The upstream separation line and, for the case of flow normal to a face ( $\theta = 0^\circ$ ), the downstream reattachment zone can be seen quite clearly.

There is a further obvious line, marked *A*, downstream of the main separation line but upstream of the block and curving round it for the case  $\theta = 0$ . This line (which also seemed to be a separation line) presumably marks the upstream extremity of the small horseshoe vortex which wraps itself around the body. This vortex only exists because of the presence of a (thin) upstream boundary layer, there being no other source of spanwise vorticity, so that presumably its physical size is determined largely by the thickness of this boundary layer. These tests were in fact made with the cube mounted on the tunnel floor and a similar experiment with the cube mounted on the base plate showed the line *A* to be much nearer the body since the upstream boundary layer was much thinner: about 2 mm thick instead of 6 mm.

For the case  $\theta = 45^\circ$  the effect of the strong vortices generated at the swept-back leading edges can also be seen quite clearly. In this case it is difficult to distinguish any region of reattachment downstream but this is not too surprising since the reattachment line must be very contorted, probably being further downstream under the strong trailing vortices ( $z/h \simeq \pm 0.5$ ) than on the wake centre-line ( $z/h = 0$ ). The mean velocity measurements certainly indicate an extremely complex wake structure of this sort (§ 4.2).

The other obvious feature of figure 3 is the concentration of vorticity just behind each vertical trailing edge of the cube; this must be associated with the 'rolling-up' of the separated shear layers but since the flow is highly three-dimensional no vortex shedding, in the usual sense, was anticipated. However, spectra from a hot wire placed just outside the wake did exhibit a distinct peak, albeit extremely weak. From a practical point of view this periodicity can probably be ignored altogether; it was certainly absent for case *B*. Such a conclusion would not necessarily be valid, of course, for bodies of higher aspect ratio (height/breadth).

## 4. Results

### 4.1. The surface pressure field

Figures 4(a) and (b) present the results of the surface pressure measurements for the case  $\theta = 0$  and both upstream flow conditions (cases *A* and *B*), whilst figures 5(a)–(c) show corresponding results for the case  $\theta = 45^\circ$ . In figure 4, for case *A* a second set of results obtained with the cube mounted directly on the tunnel floor is included and results obtained from the Engineering Sciences Data Unit no. 71016, which were collected from a variety of sources, are also plotted. All the results for case *A* are plotted non-dimensionally in the form of a pressure coefficient  $C_p$  defined by

$$C_p = (p_s - p_r) / \frac{1}{2} \rho U_r^2,$$

where  $p_s$  is the surface static pressure and  $p_r$  and  $U_r$  refer to the reference conditions, measured at the contraction exit. As explained in § 3 no attempt was made to correct  $C_p$  for blockage, but  $U_r$  and  $p_r$  represented well the conditions far upstream since they differed by less than 1% from the values obtained with the cube removed and the same total head in the working section.

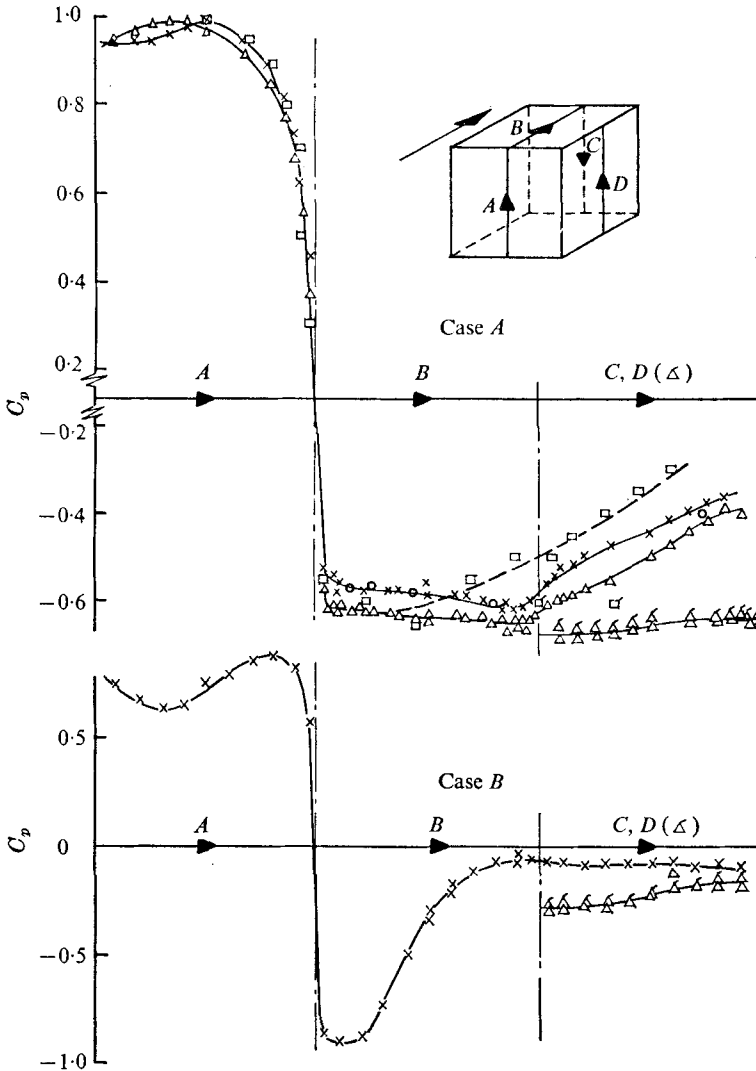


FIGURE (4a). For caption see facing page.

Consider first the results for the orientation  $\theta = 0$  (figure 4). It is clear that with the cube mounted on the tunnel floor  $C_p$  was somewhat higher in the separated regions behind the cube leading edges than the values measured with the cube mounted on the base plate. That this was solely due to the thin upstream boundary layer was demonstrated by placing a small fence, of height equal to the floor boundary-layer thickness, near the leading edge of the base plate; the pressures then rose to those measured with the cube on the tunnel floor. This is an interesting result since the floor boundary layer was originally felt to be thin enough to be insignificant: its thickness was an order of magnitude smaller than the body dimension. Clearly it is important when studying such bluff-body flows in a 'nominally' uniform free stream to ensure that the inevitable upstream



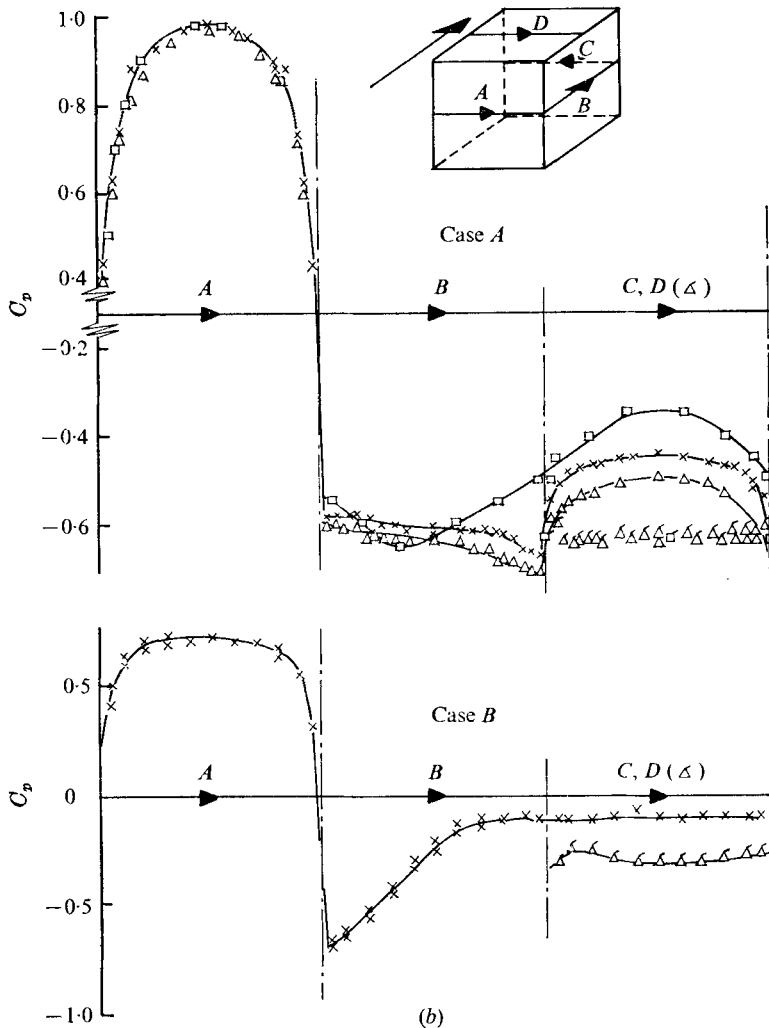


FIGURE 4. Surface pressure coefficients on the cube when normal to the incident flow ( $\theta = 0$ ).  $\times$ , cube on tunnel floor;  $\Delta$ , cube on false floor;  $\square$ , from ESDU item no. 71016. (Case A, uniform upstream flow; case B, boundary-layer upstream flow.)

boundary layer is extremely thin and if it is not to state its size. The reason for the different pressures in the two cases probably lies in the nature of the horseshoe vortex mentioned in §3. This vortex no doubt promotes extra turbulent mixing near the base of the body which entrains fluid into the base region and therefore relieves the base pressure. The larger the vortex the greater this effect and as described in §3 a thicker upstream boundary layer produces a larger vortex.

It appears, by comparison of these results (figure 4) with those in the ESDU 71016, that in most of the quoted work the 'nominally' uniform upstream flow included quite a thick boundary layer, although some of the differences may be caused by blockage effects. The other noticeable difference occurs at the vertical trailing edges of the cube, where in the present case there is an abrupt change

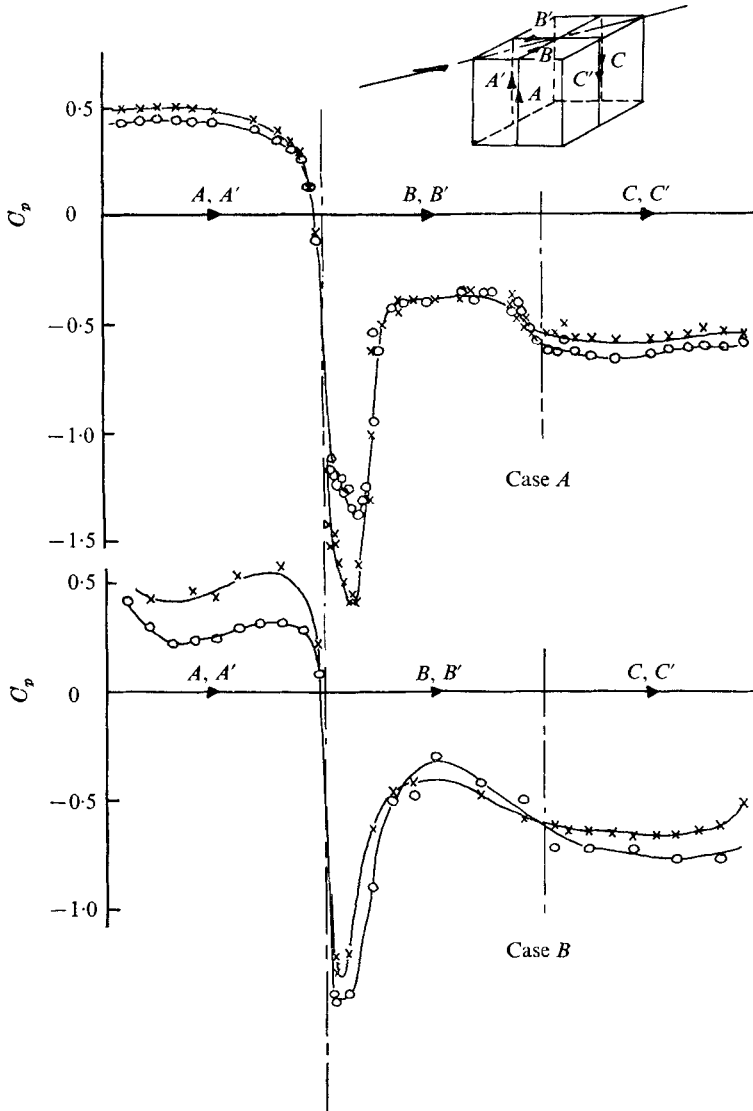


FIGURE 5(a). For caption see p. 318.

in the slope of  $C_p$ , which suggests a separating flow rather like that occurring at the leading edges and presumably caused by separation of the boundary layers growing on the faces of the cube within the cavity region. It is possible that the much smoother variation in the ESDU data results from an improper 'amalgamation' of previous results but inspection of some of the individual sources (e.g. Hamilton 1962) reveals similar trends and it seems more likely to be caused by insufficiently sharp corners. The significant effect of rounded edges has been demonstrated elsewhere (Hoerner 1965).

For case B,  $C_p$  is defined by

$$C_p = (p_s - p_\delta) / \frac{1}{2} \rho U_h^2,$$

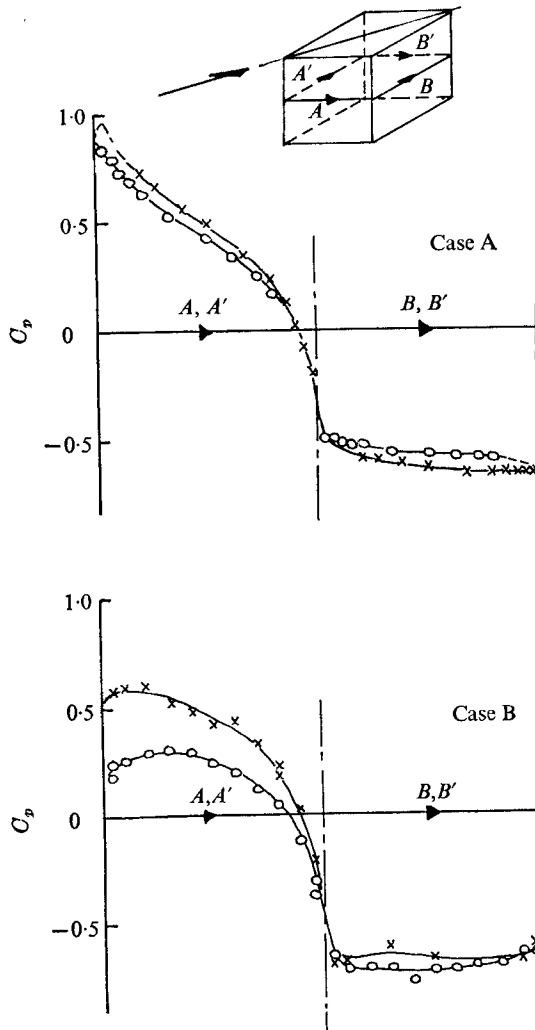


FIGURE 5(b). For caption see next page.

where  $p_\delta$  is the static pressure in the free stream above the boundary layer and  $U_h$  is the velocity in the undisturbed flow at the cube height, which seems a more meaningful velocity scale than  $U_\delta$ , the free-stream velocity. The most significant aspect of the  $C_p$  variations is that the base pressure is not very different from the free-stream static pressure and that on the sides and top of the body the surface pressure starts to rise towards this base value almost immediately after the high negative suction region round the leading edges. The negative  $C_p$  near the leading edges is not very different from that for a uniform upstream flow (case A): separation must still occur at the leading edge whatever the nature of the upstream flow. However, the recovery to a nearly zero base pressure suggests that the flow reattaches on the top and side surfaces in contrast to the uniform

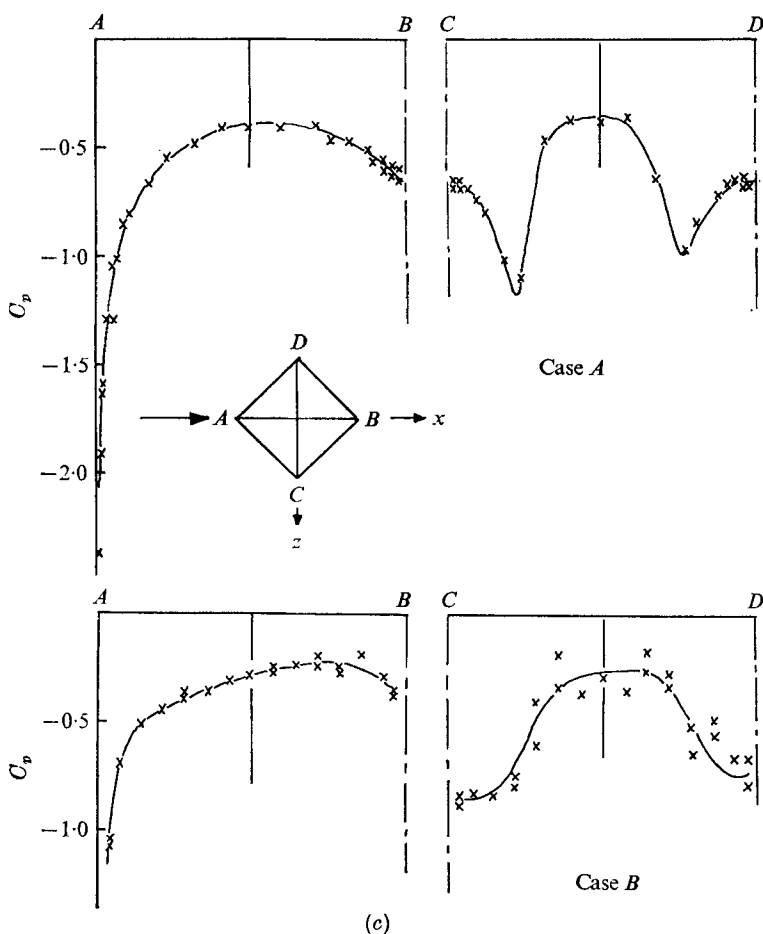


FIGURE 5. Surface pressure coefficients on the cube when at  $45^\circ$  to the incident flow.  $\times$ ,  $A$ ,  $B$  and  $C$ ;  $\circ$ ,  $A'$ ,  $B'$  and  $C'$ . (c) Top surface.

flow situation. The measurements presented in § 4.2 confirm this deduction. A feature of turbulent flow around two-dimensional square-section cylinders is that the base pressure tends to rise with increasing upstream turbulence whether or not the separating shear layers reattach (Bearman 1972; Lee 1975). Consequently a rise in base pressure would be anticipated in the present case, although reattachment of the flow on the cube surfaces may produce a greater rise than would otherwise be the case.

Consider now the results for cases  $A$  and  $B$  with  $\theta = 45^\circ$  (figure 5). A fuller discussion is deferred to § 5 but two points are worth noting here. First, there is a distinct asymmetry in the pressure distributions for both case  $A$  and  $B$ . In neither case can the stagnation point lie permanently on the vertical leading edge since, as mentioned in § 3, this would be an unstable situation. Whilst making the case  $A$  measurements, using a multibank manometer, it was clear that the actual stagnation point was switching intermittently from one side to the other and on average the flow clearly preferred one of the two states. Rotating the cube slightly in the appropriate direction was sufficient to cause the flow

to prefer the alternative state. In this case random pressure fluctuations caused by the unsteady wake would be sufficient to cause intermittent switching but in case *B* the incident turbulence is just as likely to produce the phenomenon. It may also be noticed that in the regions on the top surface where strong vortices are generated (figure 5) there is somewhat more scatter in the measurements for case *B*; this tends to mask the asymmetry in these regions.

Second, it is notable that the differences between cases *A* and *B* are much less significant than when the flow is normal to a face ( $\theta = 0$ , figure 4). The pressure distributions on the top surface, for both approach flows, are very similar to those on typical delta wings—a fact noted previously by Ackeret (1966)—and there is no possibility of reattachment on the two rear faces. The wake is, however, vastly different in the two cases as will be seen in the next subsection.

#### 4.2. Measurements in the wake

Initial exploratory measurements in the wake for case *B* and  $\theta = 0$  showed that the flow returns very rapidly to the initial upstream conditions. At  $x/h = 8.5$  there was, within the experimental accuracy, no discernible deviation of the mean velocity profile from the upstream boundary-layer profile and even at  $x/h = 4.5$  the differences amounted to only a few per cent. For the purposes of comparison it was therefore unnecessary to extend the range of measurements in case *A* beyond about eight cube heights downstream. Such an investigation would anyway pose a variety of extra problems and for a proper study of the far wake the experimental set-up would need to be considerably altered.

A comprehensive range of measurements of longitudinal mean velocity and turbulent intensity was made, including vertical profiles around the cube and on the wake centre-line for  $-0.5 \leq x/h \leq 8.5$  and spanwise profiles in the same  $x/h$  range and at various heights ( $0.3 \leq y/h \leq 1.0$ ). It would not be reasonable to present all the resulting data but in order to demonstrate the significant effects of both cube orientation and the nature of the upstream flow some typical profiles of mean velocity and turbulent intensity are shown in figures 6–11. There are a number of points worth making here before further discussion in § 5. From the measurements of mean velocity above the centre of the body (figure 6*a*) it is clear that there is no separation region above the cube at  $x/h = 0$  for case *B* and  $\theta = 0$ . In contrast to the uniform upstream flow situation (case *A*) the shear layer separating from the leading edge reattaches on to the top surface. At  $x/h = -0.25$  the thickness of the negative velocity region was found to be only about 5% of the cube height so reattachment must occur near  $x/h = -0.2$ . Similar conclusions apply to the side faces. For the case  $\theta = 45^\circ$  the differences are less significant; whatever the approach flow there is no reversed flow region above the cube on  $x = z = 0$ .

For the case  $\theta = 0$  the intensity distributions (figure 7*a*) show a peak either where there is a separated shear layer or where the flow has recently reattached (and is therefore of shear-layer character). In contrast, for  $\theta = 45^\circ$ , there is simply a monotonic rise in turbulence energy as the top of the cube is approached. The differences between the undisturbed boundary-layer values of turbulent intensity and the apparent asymptotic values above the cube are probably within the

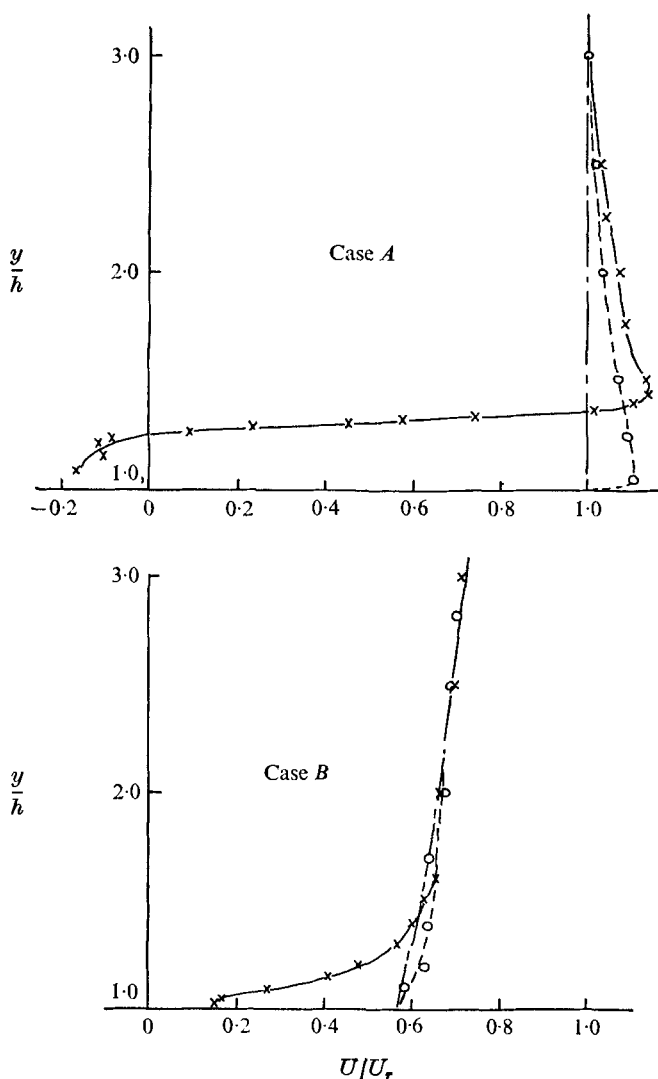
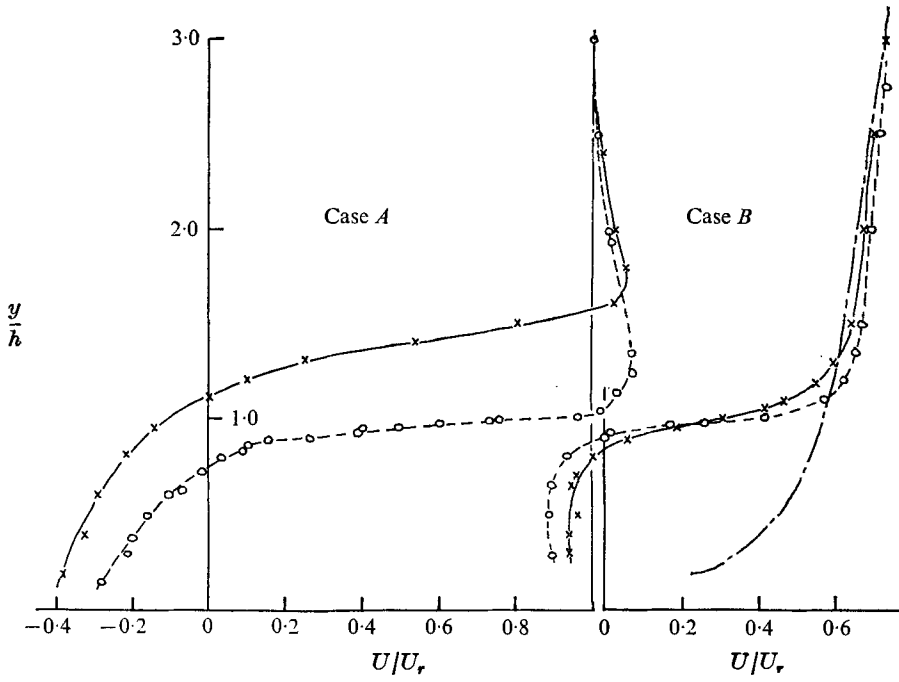


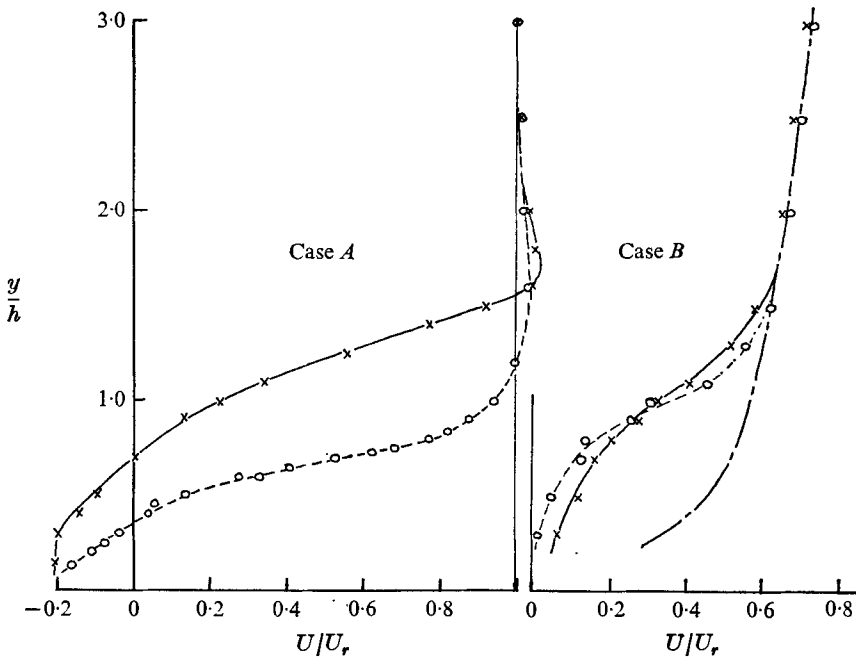
FIGURE 6(a). For caption see facing page.

combined experimental accuracy of the present pulsed-wire measurements and the original boundary-layer measurements (Castro *et al.* 1975).

In the near wake behind the body the differences between the typical mean velocity profiles on the centre-line (figures 6*b*, *c*) can be attributed basically to the position and state of the separated shear layer. For case *A* and  $\theta = 45^\circ$  there is no separation above the cube centre-line so in the wake the shear layer has originated from the 'boundary layer' which separated at the rear of the body. It is, therefore, considerably lower than in the case  $\theta = 0$ , where the shear layer has originated from the leading edge and is also somewhat thinner with a correspondingly larger peak in turbulent intensity (figures 7*b*, *c*) since it has had less time to grow. However, for case *B* the shear layer separates from the rear of the



(b)



(c)

FIGURE 6. Mean velocity variations in the wake on  $z/h = 0$ . (a)  $x/h = 0$ ; (b)  $x/h = 1.0$ ; (c)  $x/h = 2.0$ . — x —,  $\theta = 0^\circ$ ; - - o - -,  $\theta = 45^\circ$ ; ·····, upstream variation.

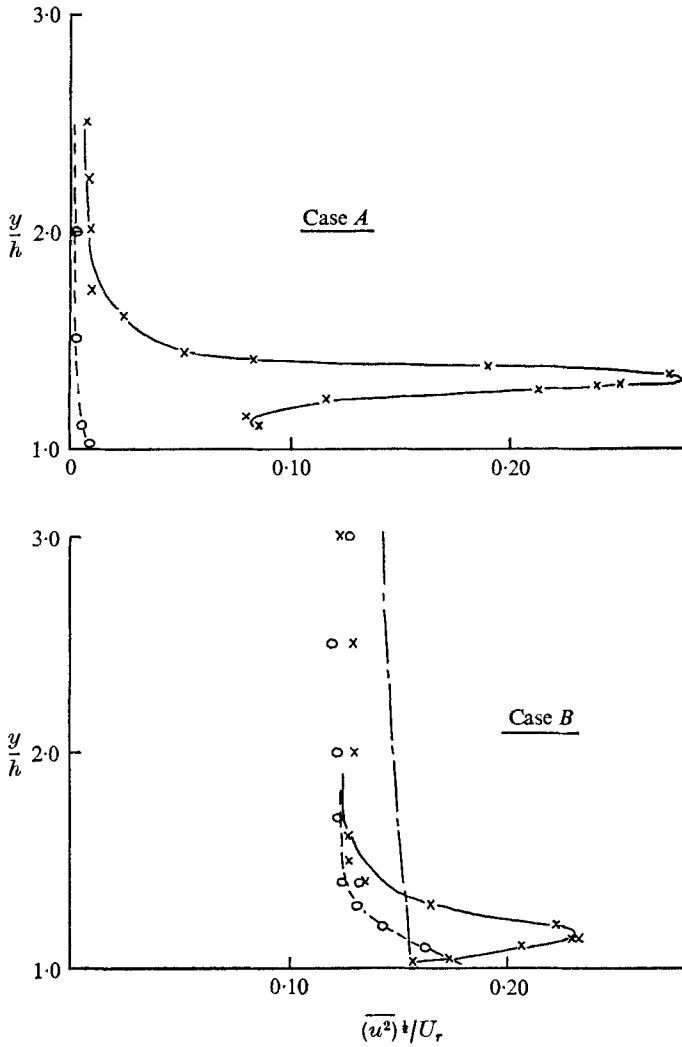


FIGURE 7(a). For caption see facing page.

body in both orientations, so that the only significant difference is in the actual shear developed. For  $\theta = 0$ , as shown above, the flow reattaches to the top surface, so that the 'boundary layer' separating at the rear edge is rather thicker than in the case  $\theta = 45^\circ$ . The actual value of the shear  $\partial U/\partial y$  at similar positions in the wake is therefore rather less.

Spanwise profiles of mean velocity at  $x/h = 2.0$  are shown for case *A* in figure 8 and it is clear that for  $\theta = 45^\circ$  the delta-wing-type vortices generated by the sweptback leading edges have a dominant influence on the flow. The axial velocity drops to a minimum, presumably near the central core of each vortex, and rises again to a maximum on the centre-line. Turbulent intensity distributions (not shown) have distinct peaks in the regions of maximum shear. Nearer the body ( $x/h < 2.0$ ) the vortices have had less time to diffuse, so that the central



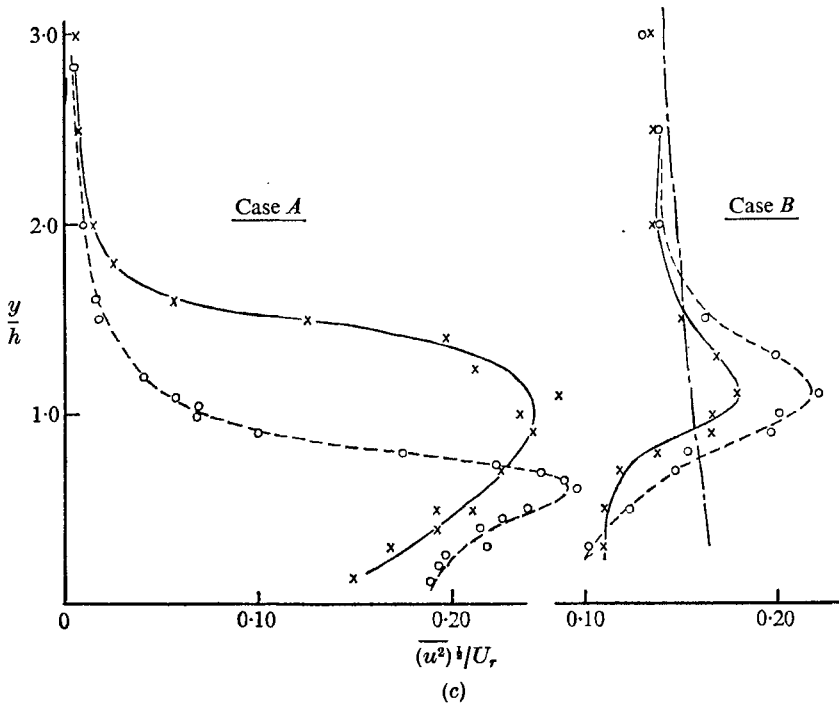
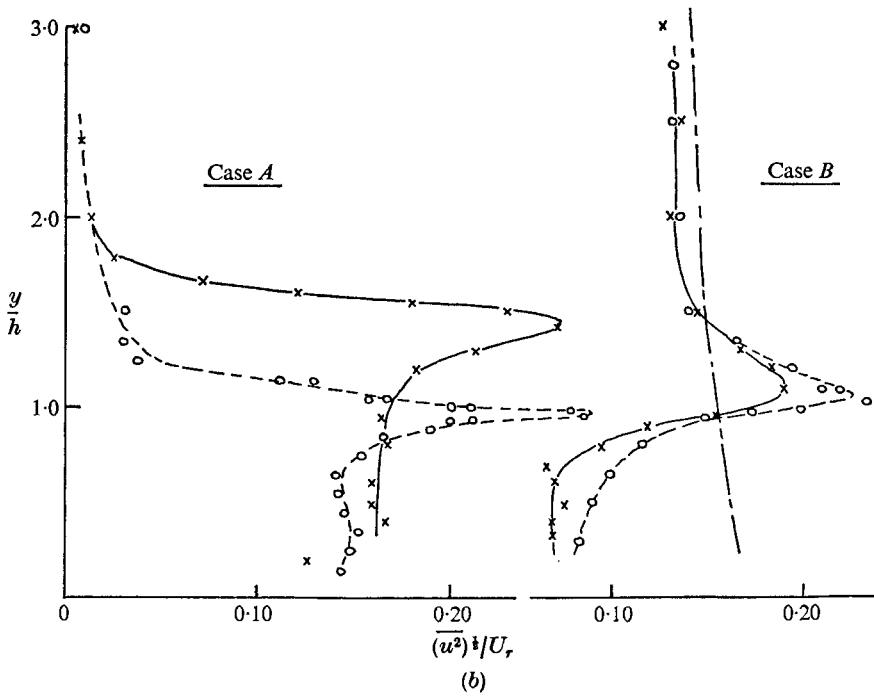


FIGURE 7. Turbulent intensity variation on  $z/h = 0$ . (a)  $x/h = 0$ ;  
 (b)  $x/h = 1.0$ ; (c)  $x/h = 2.0$ . Symbols as in figure 6.

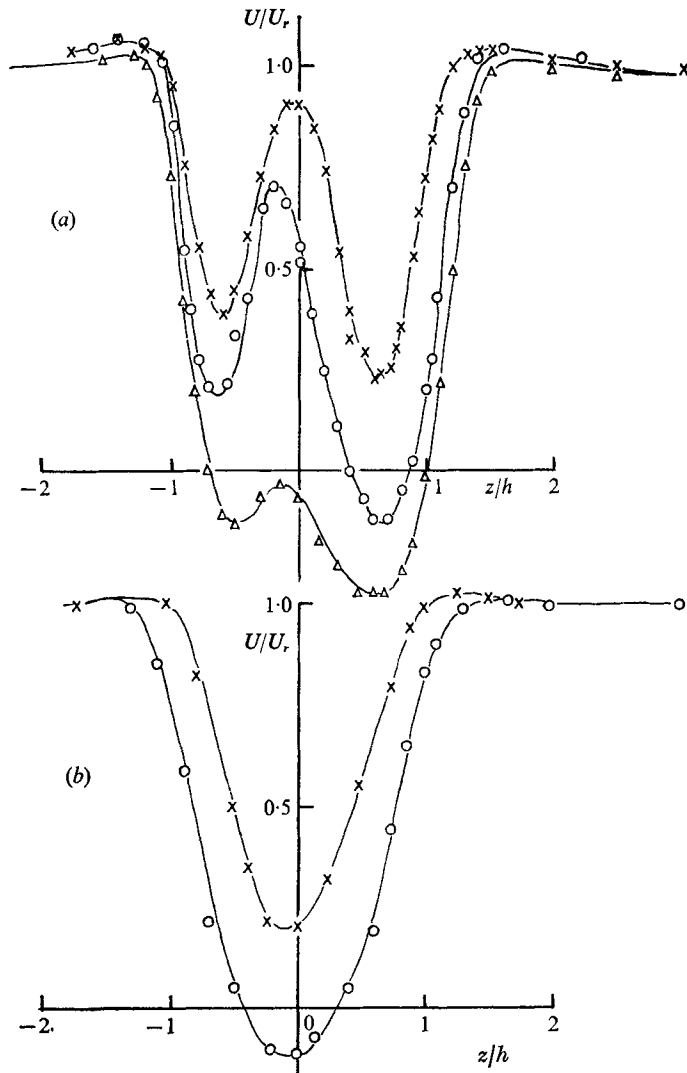


FIGURE 8. Spanwise mean velocity variations at  $x/h = 2.0$  for uniform upstream flow (case *A*). (a)  $\theta = 45^\circ$ ;  $\times$ ,  $y/h = 1.0$ ;  $\circ$ ,  $y/h = 0.7$ ;  $\triangle$ ,  $y/h = 0.3$ . (b)  $\theta = 0$ .  $\times$ ,  $y/h = 1.0$ ;  $\circ$ ,  $y/h = 0.5$ .

peak in the mean velocity variation at, say,  $y/h = 0.7$  was less pronounced. Conversely, further downstream ( $x/h > 2.0$ ) the central peak at, say,  $y/h = 0.3$  was more pronounced than the central peak at  $x/h = 2.0$ . It is evident that for  $\theta = 45^\circ$  the flow was distinctly asymmetric; the vortex on one side (positive  $z$ ) was of greater strength than that on the other side as indicated by the pressure measurements (§ 4.1).

Time did not permit many spanwise measurements for case *B* but figure 9 is sufficient to demonstrate that, particularly for  $\theta = 45^\circ$ , the flow is very different. A similar vortex system must be developed by the cube at  $45^\circ$ —the pressure measurements certainly show little difference between cases *A* and *B*

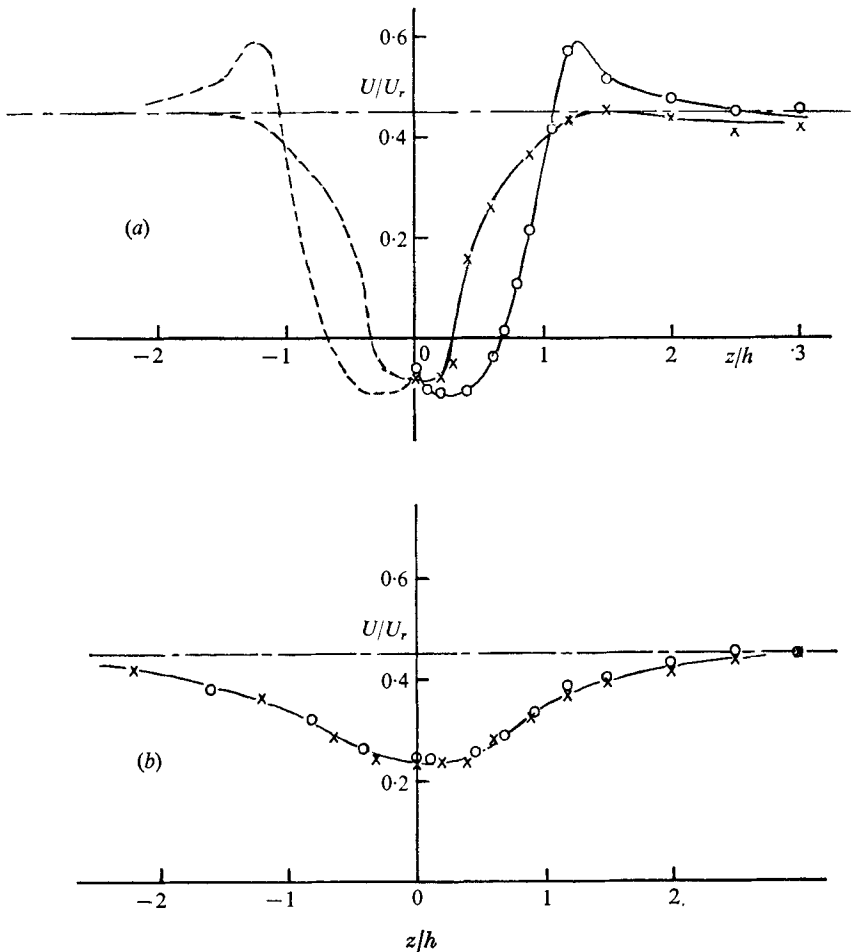


FIGURE 9. Spanwise mean velocity variations for boundary-layer upstream flow (case B). -x-,  $\theta = 0^\circ$ ; -o-,  $\theta = 45^\circ$ .  $y/h = 0.5$ . (a)  $x/h = 0.75$ , (b)  $x/h = 3.0$ .

for  $\theta = 45^\circ$ —but the complete lack of any strong vortex system at  $x/h = 3.0$  is clear from a comparison of figures 8 and 9. Although there is slight evidence of an effect at  $x/h = 0.75$  it must be concluded that the shear and turbulence in the upstream flow provide a powerful mechanism for the rapid breakdown of the vortex system generated at the body. Further discussion is deferred to § 5.

Even with the cube normal to the flow the wake must contain axial vorticity. The majority of this arises from the horseshoe vortex generated from the upstream shear (even for only a very thin upstream boundary layer) although some may possibly be shed at the top axial edges of the body. Because of the rapid disappearance of the wake in case B the only evidence of axial vorticity is indirect and arises from measurements of the diffusional properties of the wake (Robins & Castro 1977). However, figure 10 shows typical spanwise variations of longitudinal mean velocity and turbulent intensity for case A and  $\theta = 0$  measured in the 'far' wake, some body heights downstream of the separated

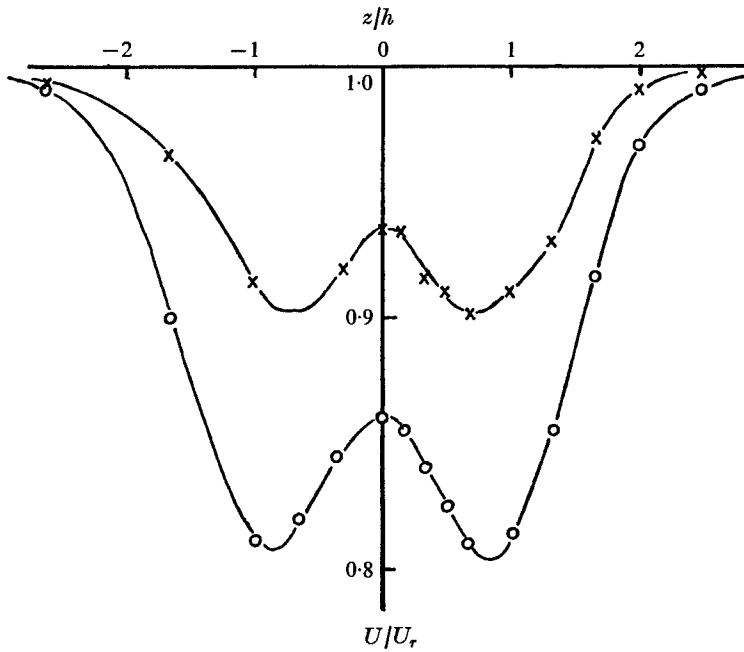


FIGURE 10. Spanwise mean velocity variation at  $x/h = 6.5$  for  $\theta = 0$  and uniform upstream flow (case *A*). — $\times$ —,  $y/h = 1.0$ ; — $\circ$ —,  $y/h = 0.5$ .

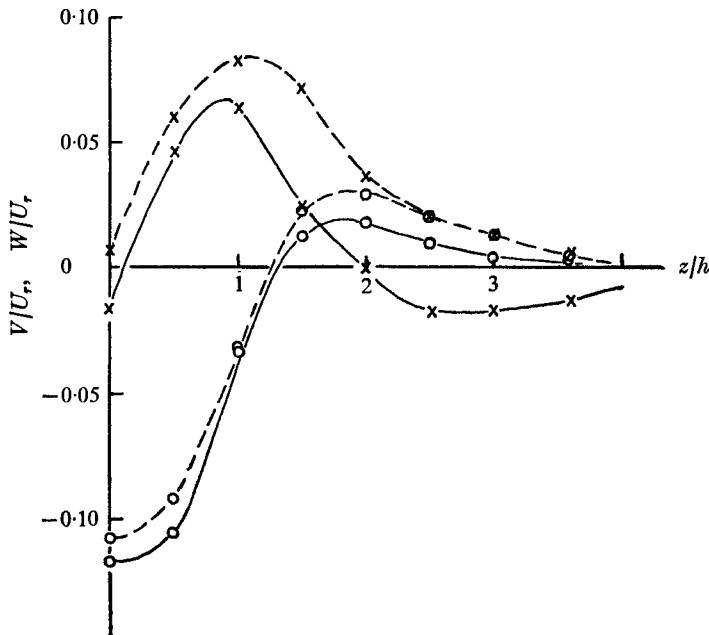


FIGURE 11. Swirl velocity components at  $x/h = 6.5$ ,  $y/h = 1.0$ . Case *A*.  $V/U_r$ : — $\circ$ —,  $\theta = 45^\circ$ ; -- $\circ$ --,  $\theta = 0$ .  $W/U_r$ : — $\times$ —,  $\theta = 45^\circ$ ; -- $\times$ --,  $\theta = 0$ .

region. It is interesting that the spanwise velocity variations have developed distinct minima away from the wake centre-line and are beginning to look like those for  $\theta = 45^\circ$  in figure 8. This first begins to occur at about  $x/h = 4.5$ , the minima gradually moving away from the centre-line in common with the peak in turbulent intensity as distance downstream increases. These effects cannot be attributed solely to vertical components of vorticity associated, for example, with the shear layers separating from the vertical, leading edges, since they are not apparent in two-dimensional wake flows. For  $\theta = 45^\circ$  measurements corresponding to those in figure 10 ( $\theta = 0$ ) show very similar variations of longitudinal mean velocity and in figure 11 direct measurements of the swirl velocities for both cube orientations are presented. It is remarkable that, despite the very different nature of the flow near the cube for the two orientations, beyond about  $x/h = 6.0$  the mean velocity variations (all three components) are all very similar. Even in uniform flow therefore, it would seem that the strong axial vorticity generated by the leading edges of the cube at  $45^\circ$  is destroyed quite rapidly, presumably by the wake turbulence and shear, whereas the axial vorticity generated by the (thin) up-stream boundary layer seems to take longer to manifest itself, possibly because some of its strength must be generated by the prolonged stretching of the vortex lines as they are swept round the body. Although this source of axial vorticity does appear eventually to affect the axial velocity field, as well as generating swirl, it too must presumably decay further downstream, though further measurements would be required to clarify the point.

## 5. The nature of the wake

Although the general problem of flow around three-dimensional surface-mounted bodies is highly complex the presence of the surface may provide constraints, which in some respects could simplify the problem, particularly for bodies of low aspect ratio (low height/breadth ratio). Vortex shedding, if it occurs at all, will not be dominant and, more significant, the scale of turbulence in the upstream flow may not be very critical. The length scales of the energy-containing motions within the boundary layer will in nearly all cases of practical significance be of the same order as the body height. Bearman (1972) and Lee (1975) have demonstrated that length scale/body height variations from about 0.5 to 5 have an insignificant effect on the pressure distributions around two-dimensional sharp-edged bodies in a turbulent stream and Laneville, Gartshore & Parkinson (1977) have concluded that similar variations have no apparent effect on the nature of the reattaching shear layers; variations of length scale/body height within a boundary layer are, of necessity, likely to be no greater than this.

There are perhaps two major parameters which affect the flow, and although they are not independent, it is possible to identify at least qualitatively the various effects that each has. First, there is the nature of the upstream flow and it seems that the important variables are likely to be the turbulence intensity and shear. Second, there is the particular shape and orientation of the body.

### 5.1. The effect of upstream flow conditions

As far as the flow close to the cube is concerned the major effect of the upstream turbulence in case *B* is to promote reattachment of the separating shear layers on the sides and top of the body (for  $\theta = 0$ ), see § 4. Such a reattachment has, in the past, been inferred from pressure measurements (Lee 1975; figure 4*a*) and occasionally demonstrated by flow visualization (Laneville *et al.* 1977) for two-dimensional bodies in free flow but the present results appear to be the first direct evidence of the effect for a surface-mounted three-dimensional body, apart from some work of McLaren (1970).

It was evident whilst making the pulsed-wire measurements in this region (figure 6*a*) that the reattachment was not intermittent and, because of the obvious practical implications, it is of value to ask under what conditions reattachment will always occur. One might expect that for some smaller value of  $\delta/h$ , i.e. a lower level of upstream turbulent energy at, say, the body height, reattachment would not occur. Indeed, Meroney & Yang (1971) studied the effects of a cube on the dispersion from point sources and in their case, where  $\delta/h$  was about four, it was not obvious from flow-visualization pictures that the shear layers reattached on the body surfaces. In an attempt to resolve this question some measurements of the variation of surface pressure on the top surface  $z = 0$  of the cube (for  $\theta = 0$ ) were made for a variety of cube heights in a variety of boundary layers in the  $0.27 \times 0.91$  m wind tunnel. Typical results are shown in figure 12 for a range of  $\delta/h$ .

Since the cubes were placed at various axial positions in the working section, the reference static pressure was in this case measured in the free stream at the cube location, rather than at the contraction exit. For the 60 mm cube this resulted in  $C_p$  values on the top surface some 15% higher (less negative) than those plotted in figure 4. Although the boundary layers were not all exactly similar it is clear from figure 12 that as  $\delta/h$  increases from 0.7 to about 0.9 a complete change in the form of the pressure distribution occurs and as  $\delta/h$  increases further the position of the maximum  $|C_p|$  moves forwards and the minimum  $|C_p|$  at the rear of the top surface falls. There is also a consistent difference in the  $C_p$  variations near the rear edge ( $x/h = 0.5$ ) for  $\delta/h$  values greater than about 2.0 and less than about 1.4. In the latter case  $|C_p|$  increases slightly as  $x/h$  approaches 0.5 and its final value is greater than the base pressure measured at the centre of the rear face (not shown) by typically a factor of two. On the other hand, for  $\delta/h$  greater than about 2,  $|C_p|$  falls all the way to the rear edge, where it is of the same order as the base pressure. Figure 13(*a*) shows the values of  $C_p$  at the rear edge plotted against  $\delta/h$ . For  $\delta/h$  greater than about 1.4 there is a slow but steady rise in  $C_p$  and it seems reasonable to suppose that in this range the shear layer separating at the leading edge reattaches permanently on the roof, the reattachment point moving forwards with increasing  $\delta/h$ , whereas at values less than about 1.4 the reattachment is intermittent. The results are somewhat more scattered in the latter range, which may be a result of increased blockage effects for this flow regime. Also plotted in figure 13 is the point deduced from the case *B* measurements described in § 4.1, and its value is somewhat

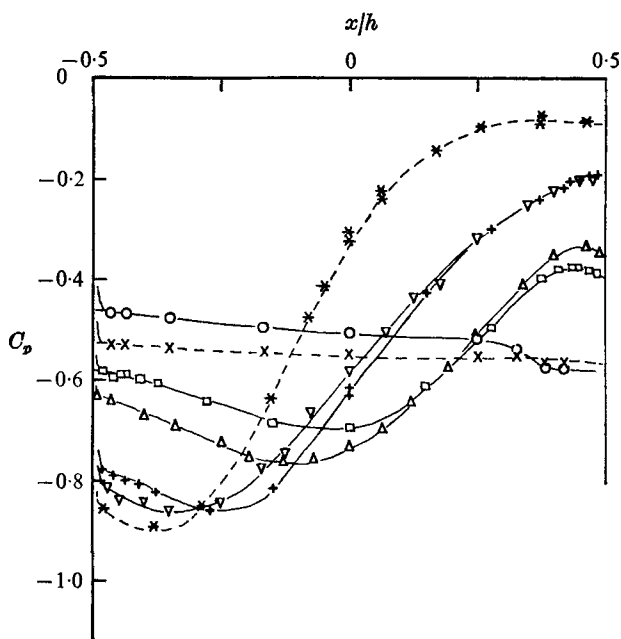


FIGURE 12. Static pressures on cube roof for various  $\delta/h$ ;  $z/h = 0$ . -x-, uniform upstream flow (case A).  $\delta/h$ : —○—, 0.583; □, 0.875; △, 1.063; +, 2.127; ▽, 8.0; —\*—, 10 (case B).

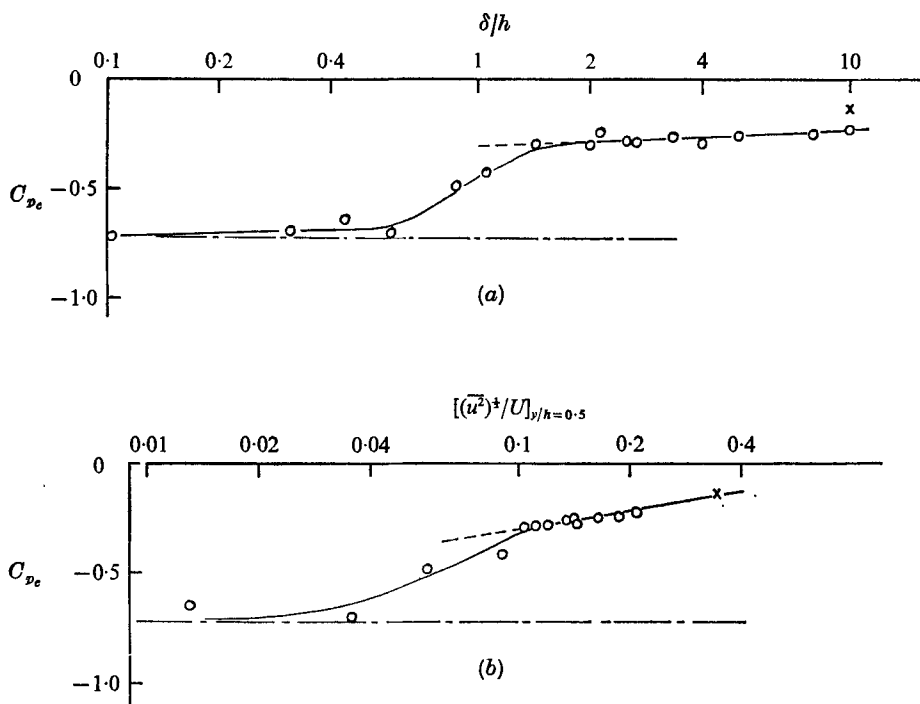


FIGURE 13. Roof pressure coefficient at  $x/h = 0.5$ ,  $z/h = 0$  vs. (a)  $\delta/h$  and (b) turbulent intensity in upstream flow at  $y/h = 0.5$ . ---, uniform upstream flow (case A); —○—, from results like those in figure 12; ×, case B.

higher than the other measurements. The turbulent energy in the upstream boundary layer of case *B* was considerably higher than in the relatively less rough boundary layers used for these later tests, and if the  $C_p$  values in figure 13(*a*) are plotted against the upstream turbulent intensity at, say,  $y/h = 0.5$  the case *B* point clearly lies on a continuation of the trend suggested by the other measurements (figure 13*b*). Laneville *et al.* (1977) have recently shown that for two-dimensional bodies in free flow it is the level of turbulent energy on the stagnation streamline which largely governs the behaviour of the separating shear layers. Similar conclusions seem to apply in the present case, so that reattachment can occur even for bodies of a height similar to the boundary-layer height. The measurements of Leutheusser & Baines (1967) and McLaren (1970) imply effects similar to those described above but quantitative comparisons are difficult because of (in the former case) lack of data on the upstream boundary-layer flow and (in the latter case) the wide range of blockage ratios. Ranga Raju, Loeser & Plate (1976) have suggested that for two-dimensional fences mounted in turbulent boundary layers the drag is a unique function of  $y_0/h$ , where  $y_0$  is the boundary-layer roughness length. However, it is clear that for bodies where reattachment is likely to occur this is not necessarily the dominant parameter.

Consider now the characteristics of the flow behind the cube. The mean velocity measurements, of which figure 6 is typical, clearly demonstrate that the size of the cavity region is markedly reduced by the presence of upstream turbulence. This can have important results for the dispersion of effluents from sources near the body (Robins & Castro 1977). These are believed to be the first reasonably accurate wind-tunnel measurements in the highly turbulent region behind a surface-mounted bluff body and, indeed, the only previous measurements, made further downstream where the turbulent energies are somewhat less, appear to be those of Counihan (1971), Lemberg (1973) and Peterka & Cermak (1977). Both intuition and theory (Hunt 1970) suggest that the maximum velocity deficit at any particular downstream location will be in some way inversely related to typical turbulent energies in the upstream flow. Table 1 shows how the distance to a 10% maximum mean velocity deficit on the wake centre-line, i.e.  $x/h$  for maximum  $\Delta U/U = 0.1$  on  $z/h = 0$ , varies with various boundary-layer parameters in these experiments.

Counihan's measurements are rather difficult to interpret since instrumentation limitations made it necessary to hold the measuring probe fixed and move the model in what appeared to be a still-developing boundary layer. However, these apart, there is a distinct trend in the various results; as the roughness length/body height ratio decreases the wake takes longer to decay as expected. The fact that the intensities at, say,  $y/h = 1.0$  do not follow (even neglecting Counihan's data) the trend in  $y_0$ ,  $n$  or the shear  $\partial U/\partial y$  at the body height suggests that one or more of the boundary layers was still developing. If it is *only* the upstream intensity which determines the decay rate, the results are difficult to interpret unless one accepts that variations in experimental set-up and the resulting measurement inaccuracies are sufficient to explain the anomalies. Certainly the difference between cube wakes in very rough boundary layers and



Author	$\sim n$ (power-law exponent)	$\frac{\delta}{U_0} = \frac{\partial U}{\partial y}$ at $y = h$	$\overline{(u^2)}^{1/2}/U$ at $y = h$	$\delta/h$	$(y_0/h) \times 10^4$	$\sim x/h$ for $(\Delta U/U)_{\max} = 0.1$
Peterka & Cermak (1977)	$\frac{1}{4}$	1.6	0.20	12	830	Very 3
Present work	$\frac{1}{4}$	1.4	0.27	10	200	rough 4-5
Counihan (1971)	$\frac{1}{7}$	0.8	0.14	8	25	Less 8
Lemberg (1973)	$\frac{1}{7}$	0.7	0.09	6	5	rough 6

TABLE 1. Approach-flow characteristics and wake lengths.

in fairly smooth (but still fully aerodynamically rough) boundary layers is clear but it is also evident that more comprehensive and accurate data are required before quantitative trends can really be established.

5.2. The effect of body orientation

Although we have only investigated the two symmetric cube orientations it is clear that the flow around the cube is highly dependent on orientation. In particular for  $\theta = 45^\circ$  the surface pressure, and hence by implication the flow near the body, is dominated by the presence of strong vortices generated by the top leading edges (§ 4). As mentioned in § 4.1, a turbulent upstream flow has remarkably little effect on the form of the pressure distribution for  $\theta = 45^\circ$ . Armit (1974) noted similar pressure distributions on a 4:1 planform model and again the presence of upstream turbulence and shear had little effect. By contrast it is evident from the results discussed in § 4 that upstream turbulence and shear can have an overwhelming effect on the decay of the strong swirling motion generated at the body. Hunt (private communication) has recently shown by theoretical considerations that the shear (more strictly  $V \partial U/\partial y$ ) over the body where the vortices are generated has a strong influence on the axial velocity field via the swirl velocity components. It could be argued that for the uniform upstream flow case this shear will in fact be greater, so that the effects on the axial velocity are large compared with the case *B* situation: compare figures 8 and 9. However, upstream turbulence will cause a more rapid decay of the swirl components and in the present work the swirl certainly seemed to decay as rapidly as the rest of the wake. Counihan (1971) also found no evidence of strong swirl downstream. Lemberg (1973), whose boundary layer was somewhat smoother than either of these, does report swirling motions some way downstream but it is not clear experimentally whether these differences are attributable to the decreasing shear or the decreasing turbulent energy in the three cases (table 1).

Peterka & Cermak (1977) do not present any measurements for a cube at  $45^\circ$  but found significant swirl effects as far as 80 body heights downstream for a body with a planform aspect ratio of about 3.2:1, even though their boundary layer was comparable to that in the present work (table 1). In this case one of the trailing vortices was of course much stronger than the other, so that the tendency for the vortex system to move downwards towards the surface was probably rather weak. The effect of the axial vorticity presumably lasts rather longer than it would if vortex pairs moved nearer the wall region, where the local turbulence

levels are much higher. This may not be a totally adequate explanation but what does seem clear is that the actual planform shape of the body can have a considerable influence on the effectiveness of upstream turbulence and shear in promoting decay of the swirling motions.

Whether or not the swirling motions are rapidly dissipated there will still be, at least for  $\theta = 45^\circ$ , considerable downwash on the centre-line above the cube itself and in fact the diffusion studies reported by Robins & Castro (1977) substantiate this conclusion: maximum concentrations of neutrally buoyant gas emitted from a source above the centre of the cube were always greatest for the orientation  $\theta = 45^\circ$ . For  $\theta = 0$  there is still some downwash on the wake centre-line for a uniform upstream flow (see § 4.2) and there is no reason to suppose that the sign of the axial vorticity should change with the addition of upstream turbulence and shear, particularly as shear will tend to enhance any vorticity shed from the side edges. Colmer (1970), however, deduced from measurements behind a hangar at RAE Bedford that the swirling motions produced upwash on the wake centre-line and certainly for bodies of different aspect ratio there could be some upwash in the near wake.

The above remarks imply that extrapolation of the flow details, even qualitatively, from one body shape to another or, more obviously, one body orientation to another, should be attempted with caution. Since the swirling motions can be of considerable practical importance this aspect of the flow clearly merits further study.

### 5.3. Theoretical/numerical methods

No doubt because of the extreme complexity of the problem there have been very few attempts made either to describe the flow theoretically or to use numerical calculation techniques to predict it. Counihan *et al.* (1974) have formulated a theory which relates the forces on the body to integrals of the wake velocity for two-dimensional surface-mounted bodies within turbulent boundary layers. This work has been extended to three-dimensional bodies (Hunt 1970) and Lemberg (1973) has described some theoretical work which, whilst being similar in many ways to that of Hunt, differs in certain important respects. The basic assumption embodied by all this work is that the velocity deficits should be small, so that strictly the theories are valid only some way downstream. Hunt made the further radical assumption that the shear stresses in the wake can be described by constant eddy viscosities related to the properties of the incident boundary layers. It is not appropriate here to include a quantitative comparison between the measurements described earlier and these theories. There are anyway two major difficulties in making the comparison. First, we have shown that in our case of a comparatively rough wall boundary layer the body wake decays extremely rapidly, so that any similarity region predicted assuming a small velocity deficit would presumably be very small indeed; there are insufficient measurements to enable really useful comparisons. Second, the swirling motions were ignored entirely in the theoretical descriptions of the flow, so that for some cases ( $\theta = 45^\circ$  in particular) the theory cannot be expected to be adequate.

The other possible approach is to use computational methods like those developed by Spalding and his colleagues at Imperial College (e.g. Gosman *et al.* 1969). Calculations of bluff-body flows using these techniques with even one-equation turbulence models have not as yet been reported and there are obvious difficulties, in addition to the computer time required even for two-dimensional flows. The most serious is the inadequacy of any turbulence model yet developed in dealing with the effects of extreme distortion on shear flows. High curvature of the separating shear layers is an integral part of bluff-body flows. Recent work by the authors has indicated that even fairly sophisticated turbulence models are simply not capable of predicting the flow behaviour with any accuracy. Even gross features like the length of the cavity region behind the body are not always correctly predicted.

Validation of both these approaches must, in the last resort, depend on accurate experimental data and the lack of such data undoubtedly hampers the development of these methods.

## 6. Conclusions

Measurements in the wake of a surface-mounted cube in uniform and turbulent upstream flows, coupled with pressure measurements on the cube itself, have led to a fairly clear picture of the nature of the flow. For the flow normal to the front face pulsed-wire measurements in the reversed flow region directly behind the body have indicated that the addition of upstream turbulence and shear considerably reduces the size of this cavity zone. Unlike the case of uniform upstream flow the separating shear layers reattach to the body surfaces and measurements for a variety of cube size/boundary-layer height ratios have demonstrated that this reattachment occurs even for cube heights larger than the boundary-layer height, although in this case the reattachment is probably intermittent. Comparisons with previous work indicate that the decay of the velocity deficits in the wake is a strong function of the upstream turbulent intensities as implied by the theory of Hunt (1970). In the present case, where the upstream boundary layer was a simulation of a fairly rough 'suburban' type atmospheric boundary layer, the wake decayed completely within about six cube heights downstream.

In the case of flow approaching the cube at  $45^\circ$  the measurements indicate that in uniform flow the near wake and pressure field are dominated by strong vortices shed from the top edges of the body, very similar to the vortices shed by a delta wing. This vorticity has a marked effect on the axial component of velocity but appears to decay quite rapidly, so that, remarkably, all three velocity components beyond about six cube heights downstream are very similar to those behind a cube normal to the flow. The axial vorticity associated with the horseshoe vortex begins to manifest itself in this region. The addition of turbulence and shear almost entirely eradicates the effect of the vortices of delta-wing type on the axial velocity component and although there is indirect evidence that the downwash behind the body is greater in the  $\theta = 45^\circ$  than in the  $\theta = 0$  orientation it is clear that the vorticity decays almost as rapidly as

the rest of the wake. For lower upstream intensities, different body planform shapes or taller bodies, the rate at which the velocity deficit and/or the swirling motions decay can be very different and further experimental work is necessary to quantify these differences. Theoretical and numerical methods, which could eventually help considerably in determining the effect of the various parameters governing the flow, are in only preliminary stages of development and there is a need for more basic data with which to assess the alternative techniques. It would be helpful to have some experimental information about the separate effects of turbulence and shear. Such investigations have been made only for (mostly two-dimensional) bodies in free flow. Whilst it is felt that the present work has highlighted some of the important effects of the upstream boundary layer on the flow around a surface-mounted body it is recognized that only one body shape has been investigated. Much remains to be done and this sort of study would appear to be a vital half-way stage between the great number of full-scale and model *ad hoc* investigations on the one hand and the very basic and more classical turbulence studies on the other.

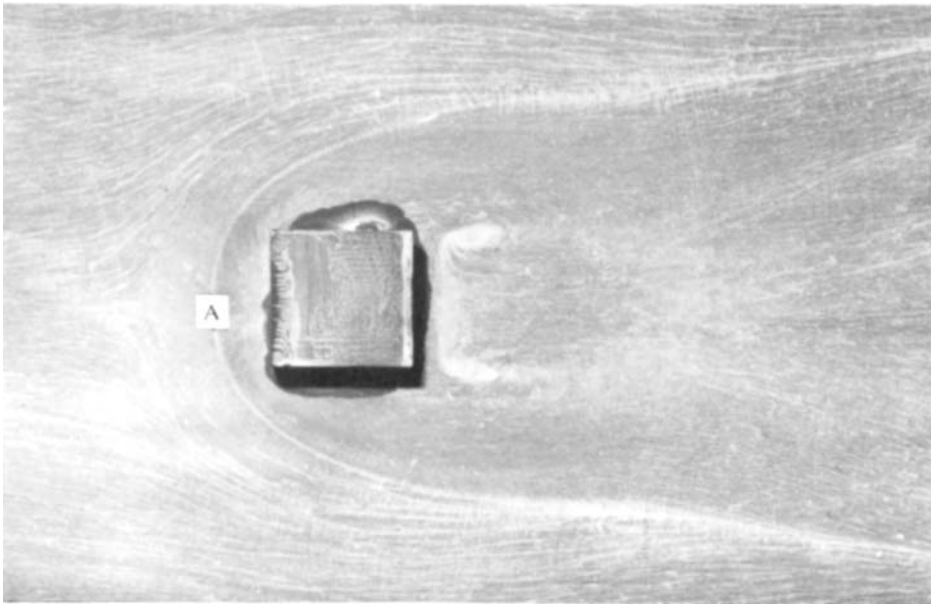
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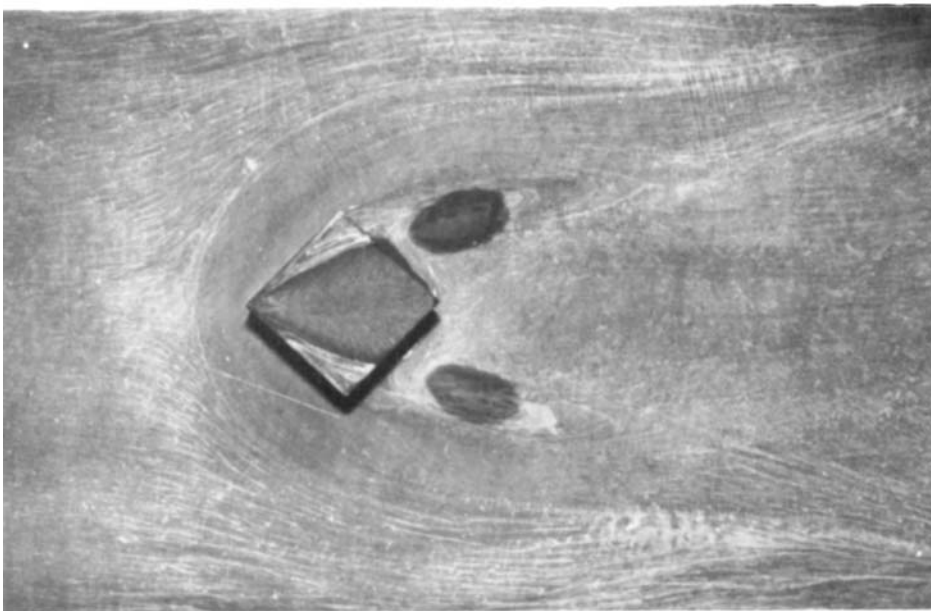
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**(a)**



**(b)**

FIGURE 3. Surface flow patterns. (a)  $\theta = 0^\circ$ ; (b)  $\theta = 45^\circ$ .